WESLEY COLLEGE
By daring & by doing

YEAR 12 MATHEMATICS METHODS

Differentiation Techniques & the Exponential Function

Test 1

Name: _____ Marks: /35

Time: 45 minutes

Calculator Free (25 marks)

1. [7 marks]

Differentiate the following functions and simplify:

a) $y = (1+3x^3)^5$ [2]

b) $y = \sqrt{\pi}\, e^{x^2+1}$ [2]

c) $y = (1-x^2)e^{4x}$ [3]

2. [5 marks]

a) Consider $f(x) = \frac{(x-2)^2}{e^{x-2}}$, clearly show that $f'(x) = \frac{-x^2+6x-8}{e^{x-2}}$ [3]

b) Determine the x-ordinates of the point(s) where the gradient of the curve is zero. [2]

3. [3 marks]

Determine the equation of the tangent to the curve $y = 3x^2 + e^{2x} + 3$ at the point $(1, 6+e^2)$.

4. [3 marks]

The curve $y = a\sqrt{x} + 3x$ has a gradient of 4 when $x = 1$.

Calculate the value of 'a'.

5. [4 marks]

If $z = 6 - x^2$ and $y = \sqrt{z}$ determine:

a) $\frac{dz}{dx}$ [1]

b) $\frac{dy}{dz}$ [1]

c) $\frac{dy}{dx}$ [2]

6. [3 marks]

Given $y = x + \sqrt{x^2 - 4}$ show that $\frac{d^2y}{dx^2} = \frac{-4}{\left(\sqrt{x^2 - 4}\right)^3}$

Calculator Section **(10 marks)**

7. [6 marks]

The temperature, $T\,^{o}C$, of a bronze casting t seconds after being removed from a kiln was modelled by $T = T_0 e^{-0.0034t}$ for $0 \le t \le 800$.

a) How long, to the nearest second, did it take for the initial temperature of the casting to halve? [2]

b) Determine the initial temperature of the casting, given that it had cooled to $787\,^{o}C$ after one minute. [2]

c) Can the above rate of change model be used to calculate how long it takes the temperature of the casting to fall below $40\,^{o}C$? Explain your answer. [2]

8. [4 marks]

The rate of decay of a radio-active material is proportional to the amount present

i.e. $\frac{dM}{dt} = -kM$ where M is the amount of radio-active material in grams and t is in years.

Given that it takes 100 years for ten grams of the materials to decay to eight grams, determine:

a) the mass present after 50 years, if ten grams were originally present

b) the material's half-life.